

Eratosthenes Project

This is an introduction to Teacher's Guide: it first recounts the observations made by Eratosthenes, his hypotheses and conclusions, followed by an overview of the way these can be put to use in class through easily-organized activities.

This project exists since September 2000, it has already helped thousands of pupils throughout the world to measure the circumference of Earth just as a man named Eratosthenes did, more than 2200 years ago. The Teacher's Guide will tell you more about this subject, but we are going to tell you in a few words the principle of that experiment:

Put a staff to the sun, measure its shadow when the sun is at its highest point, deduce the angle of the solar rays compared with the vertical staff, then exchange your results with another class under another latitude. Then, a few geometrical drawings and a rule of three give you the length of earth's meridian.

An interdisciplinary project

Many subjects will be seen, often in a ludic point of view, in order for the pupils to acquire very different knowledge (most are linked to the latest school programs).

- History and geography: We begin with ancient Egypt to explain who has been Eratosthenes, in his time and place, and at the end of the project, the pupils will have to look for points on the globe and find where are located their correspondents.
- Astronomy: the shadow of a simple staff will show the trajectory of the sun during the day and see when it is at its highest, then see how this course changes with the seasons.
- Physics, of course: light and shadow are at the heart of the project, and experiments will be done on the field, and then simulations in the classroom to reproduce what has been observed.
- Technology also, since the instruments will have to be made and adjusted: gnomons (primitive sundials), plumb line, air level, set square, quadrants, will be designed, made, tested and adjusted by the pupils.
- Mathematics, of course, especially geometry, for the pupils will have to use parallels, angles, triangles, circles, length differences...
- Oral as well as written language, since it will be used in every activity, especially for experimental process, used by *La main à la pâte*: the pupils will make hypothesis, design experiments, make observations and then give conclusions, either orally or in a day-to-day log.
- Information and communication techniques: thanks to the Internet, the pupils will make documentary researches, and discuss with other pupils to compare and exchange the results of their measures and calculations.
- Plastic arts, since this project gives the opportunity to show one's creativity: drawings showing Eratosthenes history, cartoons, models of the experiments, calligraphy to reproduce hieroglyphs and Greek alphabet...

Adapt your course

The course given in the project is an "ideal" one, and you can adapt it anytime according to numerous contingencies: age, level and motivation of your pupils, the size of the group, the time you want –or can- give to this project, as well as the hazards of meteorology... You will also use the many different answers given by the pupils and their suggestions that will sometime unexpectedly change the course of things.

You can then shorten your course, but your "minimal" course will have to keep the five following steps:

1. Show the curve of earth's surface and the parallelism of solar rays.
2. Observe the evolution of the shadow of a staff and deduce the course of the sun.
3. Discover the moment of solar midday (the time when the shadow is at its shortest).
4. Use a gnomon to deduce the angle of solar rays compared to the vertical.
5. Use the survey made by a correspondent and locate the two partners on Earth, to estimate the length of earth's meridian.

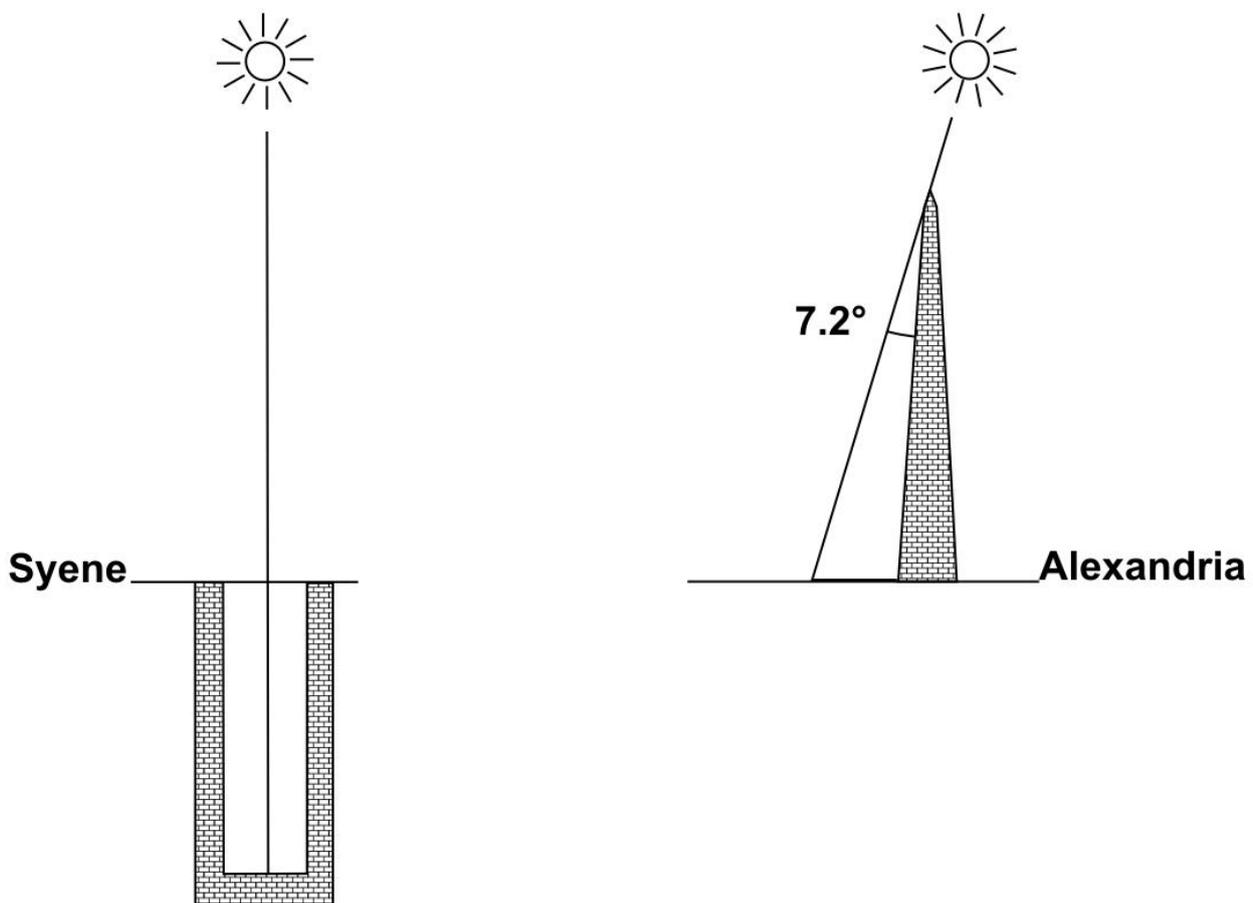
One final word about the necessary material: you'll see that it is very simple and cheap, because it is very common (Bristol, cardboard, tracing paper, screw, small boards, strings, electric lights, ball, globe...). You'll find the list at the beginning of each of the five sequences of the teaching module.

1 - The observations made by Eratosthenes

In 205 B.C., the Greek astronomer Eratosthenes, at the time Director of the Great Library of Alexandria in Egypt, proposed a purely geometrical method to measure the length of the Earth's meridian (circle passing through the poles).

He started by using the observation of shadows made at two different places, Alexandria and Syene (now Aswan) distanced approximately 800 km apart (distance estimated in relation to the time taken by a caravan of camels to connect the two towns !) at the time of the Summer solstice and at noon local solar time.

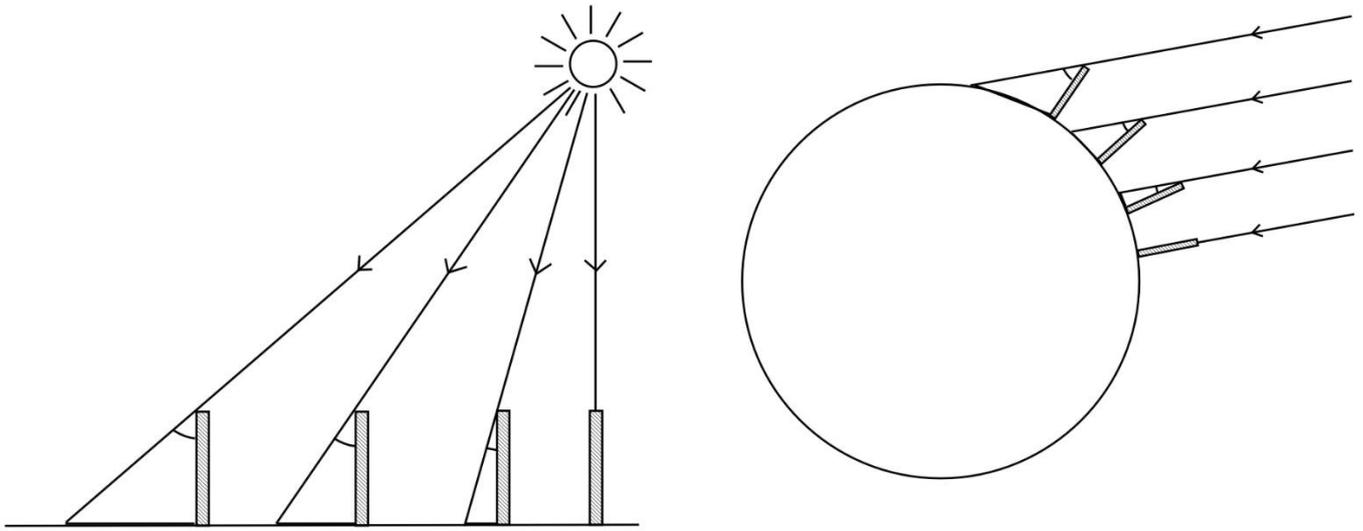
On that date and at that precise time in the northern hemisphere, the Sun reaches its highest position in the year above the horizon. However, Eratosthenes noticed differences from one place to another.



In Syene (approximately situated on the tropic of Cancer) the Sun is at the vertical, so much so that its rays reach the bottom of a well: and the shadows of vertical objects are perfectly centred round them.

In Alexandria, on the other hand, the Sun is no longer at the vertical, and these same objects have a very shallow offset shadow. Eratosthenes set about measuring the shadow of an obelisk whose height he knew, and used this information to deduce the angle of the sun's rays from the vertical: he found 7.2°

On the basis of these observations, two hypotheses lay before him:

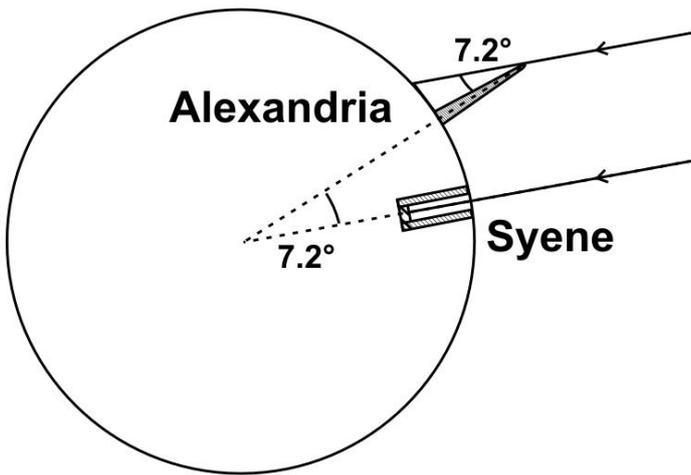


The Earth is flat, but in this case the Sun would be sufficiently close for there to be a significant divergence in its rays reaching distant objects: since objects of identical length have shadows of different length and no shadow at all when vertically underneath the Sun (zero angle).

The Earth is not flat, but has a curved, and perhaps even a round surface. Only, the same results can be obtained with sun rays which are all parallel: this implies that the Sun is sufficiently far away, very, very far away...

Eratosthenes opted for the second hypothesis. Indeed, the Ancients had already suspected that the Earth was not flat, on the basis of various observations seemingly providing evidence that its surface was somewhat curved: navigators perched on the top of their main mast are the first to perceive the distant coastline; observers on top of a cliff have a longer view of ships moving towards the horizon than observers on the beach; the pole star is not at the same height above the horizon in Greece as in Egypt; finally during eclipses of the Moon, the shadow of the Earth projected onto the Moon shows a circular section.

Convinced that the Earth is round, our genius Eratosthenes set about tracing his famous "amazingly simple" geometrical figure, which he used to calculate with ease the length of the Earth's meridian! Look for yourselves:



If the Earth is round, by extending the vertical in Alexandria (the obelisk) and the vertical in Syene (the well), these two verticals should by definition meet at the centre of the Earth. Also, Eratosthenes knew that the town of Syene being situated directly South in relation to Alexandria, the two cities were situated on approximately the same meridian. Since the sun's rays are indeed parallel, the angle formed by the two verticals at the centre of the Earth must therefore be identical to the angle he measured with the shadow of the obelisk (7.2°).

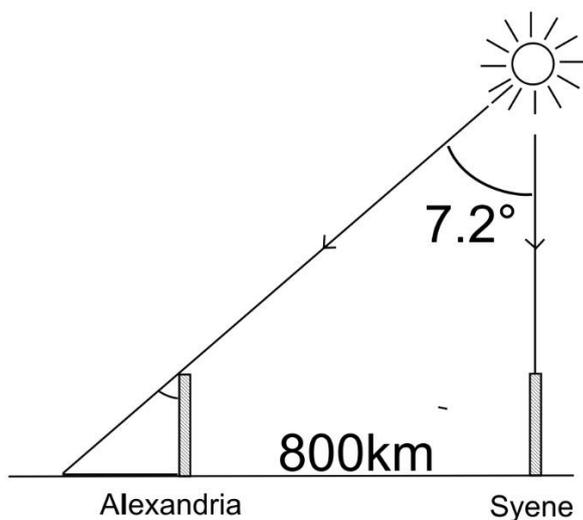
The proportion of this angle in relation to the 360° of a circle is the same as the proportion of the distance separating the two cities (approximately 800 km) relative to the circumference of a circle (in this case the Earth's meridian). The rest you can guess: 360° divided by 7.2° gives 50, and 800 km multiplied by 50 indeed gives 40 000 km (a length which was found again later but using other methods).

Angle (°)	Distance (km)
7.2	800
360	circumference



$$\text{Circumference} = 360 \times 800 / 7.2 = 40\,000$$

For the "maths experts"



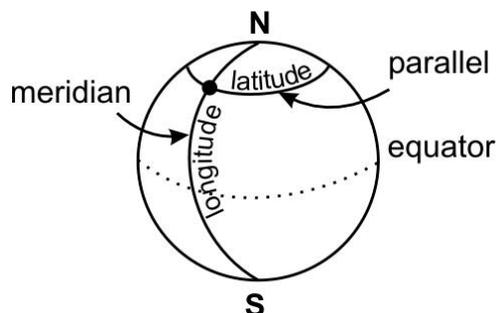
As already mentioned above, the observations made by Eratosthenes could meet the first hypothesis, i.e. the earth is flat and the Sun is very close. Some of the data provided by this brilliant genius even enable us to calculate with accuracy the distance at which this Sun would then have been.

In this case, the tangent of the angle of 7.2° would be equal to the ratio of the 800 km separating Syene from Alexandria in relation to the distance separating the Earth from the Sun:

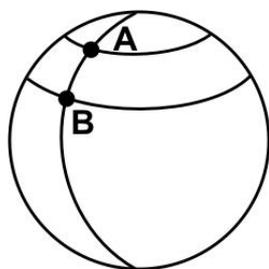
The distance of the Sun would then be found as: $800 \text{ km} / \tan 7.2 = 6500 \text{ km}$ approximately from the Earth (that is the value of the Earth's radius), which is an extraordinarily close distance, since it is known today that our Sun is about 150 million km away!

2 - Adapting an experimental project for the class

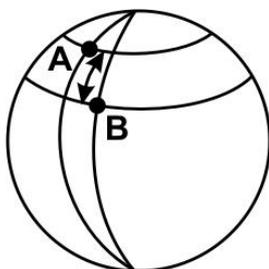
You will be conducting this experiment paired with another class (whose particulars we will be forwarding to you) but you will not need either an obelisk or a well! All you will each need is a simple vertical stick, preferably of identical height to simplify comparisons of shadow measurements.



It will not be necessary either for one of the two partners to be positioned on the tropic of Cancer! It need only be located at a distinctly different latitude to the other.



If the two schools are located on more or less the same meridian, well and good... Otherwise there will be no problem since each one in turn will "see things differently" with noon-time being different on each one's meridian!



It is not the number of kilometres between your two schools which will be taken into account, but the shortest distance separating the two parallels giving your latitudes (you will see that this distance is easy to calculate).

No need either to wait for the Summer solstice before taking your measurements! Any day of the year may be used provided that it is the same day chosen by your partner: it would be wise therefore to come to agreement with your partner and to repeat the "practical" on a certain number of days ... As for each partner's determination of midday at local solar time (different from one place to another and on every day of the year) no problem either: all you need do is to identify the shortest shadow during the half hour either side of 13 h winter time : kid's stuff... Provided of course that the Sun makes an effort!

3 - Example of two classes located in Lafrançaise (France) and Meerut (India)

Thursday 10 February 2011.

The coordinates of the two towns:

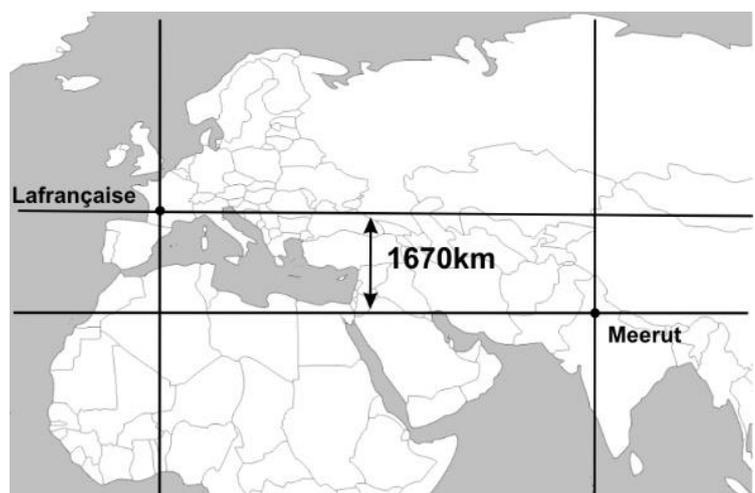
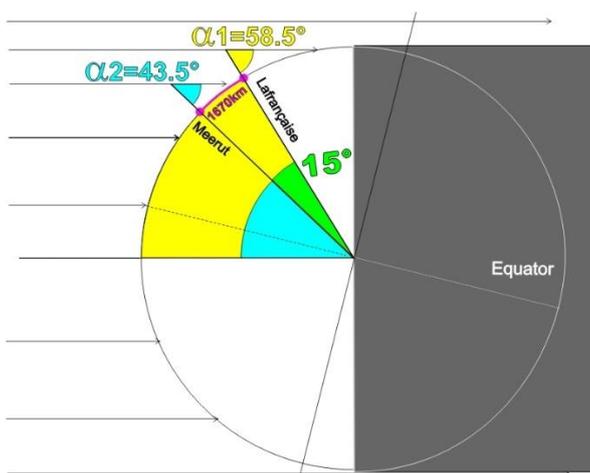
Lafrançaise: latitude $44^{\circ}08'N$, longitude $1^{\circ}15'E$

Meerut, latitude $29^{\circ}00'N$, longitude $77^{\circ}42'E$.

With these measurements the children are able to calculate the angle of the Sun's rays (at midday local solar time), accurate to half a degree, using a very simple geographical figure

For Lafrançaise they find $\alpha_1 = 58.5^{\circ}$

And for Meerut $\alpha_2 = 43.5^{\circ}$.



But how is it possible, using these two angles to calculate the famous alpha angle?

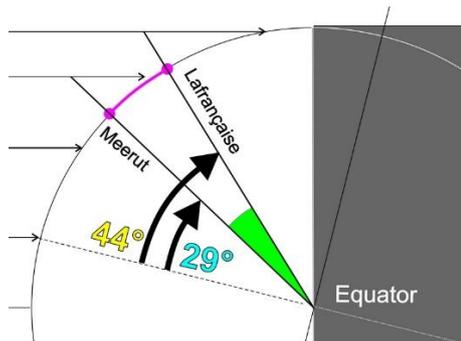
You need only subtract alpha angle 2 from alpha angle 1, which gives 15° .

Since the two partnered schools are not positioned on the same meridian, the shortest distance must then be determined between the parallel of Lafrançaise and the parallel of Meerut. This is very easy: on a map, a careful tracing is made of the two parallels and, using the map scale, the distance between them is calculated. Here the value found is in the order of 1670 km.

We now have in hand the two elements needed to calculate the Earth's meridian using "Eratosthenes' method": the alpha angle of 15° and the distance of 1670 km between the two parallels. The proportion of a whole circle relative to the result of 15° being 24 times (360° divided by 15°), the meridian is calculated by multiplying 1670 km by 24, which indeed gives 40080km....An efficient method, provided that the measurements are as precise as possible (especially if the latitudes are nearer to one another !).

Angle (°)	Distance (km)
15	1670
360	circumference

$$360 \times 1670 / 15 = 40\ 080$$



It is interesting to note, as shown in figure, that the difference between the two latitudes gives the famous alpha angle immediately! With our two partnered schools, this gives: $44^{\circ}08' - 29^{\circ}00' = 15^{\circ}08'$. We note that the measurements made by the children are very accurate since our school friends deduced an angle of 15°, which was therefore very close.

We stress the fact that the "direct method" for calculating the alpha angle using knowledge of the two latitudes must absolutely not be given to the children at the outset! On the other hand, it could be useful to them, at a later stage, to detect any possible errors in their measurements...

4 - Steps for implementing the project

In compliance with the principles of La main à la pâte, you must ensure that predominance is given to pupil reflection: you must encourage them to formulate assumptions which they can verify later by thinking up appropriate experiments. Each pupil must have an exercise book in which they can note their own research using drawings or short sentences. This book will also be used to record group work and group results. This will assist you in checking the proper understanding of the work conducted in class and to follow through the progress made by each one of your pupils.

Here then are the different steps we propose:

1/ The first sequences are placed on-line, opening of a distribution list for those schools taking part in the project. The scientists and teaching specialists will be entered into this list and will answer any queries you have.

2/ When you subscribe to the project, you are automatically added to the mailing list of the Eratosthenes's project, so as you can easily communicate with other schools involved in the project. You will also get a password that gives you access to a workplace.

This workplace will allow your school:

To login to record and look at the results of your measurements on the website of the project;

To access to coordinates of all schools involved in the project,

To get their measures and see their locations on a map of the world.

3/ All year long, schools record their measures in the workplace of the project. Simultaneous measures can be planned using the mailing list. (September and March equinoxes, December solstice)

4/ On June, 21st, schools reproduce together the historical experiment that allowed Eratosthenes to measure the size of the Earth over 20 centuries ago !